Univ. of Houston

## 11/13 - Chapter 18

## Last Lecture

Cost of capital

$$
\begin{aligned}
& \mathrm{k}_{\mathrm{c}}=(\mathrm{D} / \mathrm{V}) \mathrm{k}_{\mathrm{d}}+(\mathrm{E} / \mathrm{V}) \mathrm{k}_{\mathrm{e}} \\
& \mathrm{~V}=\mathrm{E}+\mathrm{D} \\
& \mathrm{k}_{\mathrm{c}}=\text { cost of capital = required rate of return = discount rate. }
\end{aligned}
$$

## This Lecture

In this class we answer the following question: How do we determine the cost of debt? Or how do investment banks determine the price (coupon) of a bond?

In Chapter 18, we will focus on two long term financial instruments: bonds and swaps.

## Chapter 18 - Long term financing

We will focus on two long term financial instruments:

- Bonds
- Swaps (currency)


## 1. Bond Market

The bond market (debt, credit, or fixed income market) is the financial market where participants buy and sell debt securities, usually bonds. Governments and agencies, Corporations (banks and non-banks), Municipalities \& Individuals (mortgages) are the main participants.

Size of the world bond market (2012 debt outstanding): USD 78 trillion, or 110\% of World GDP (size of world equity markets capitalization USD 53 trillion).

- U.S. bond market debt: USD 33 trillion.
- Japan bond market debt: USD 14 trillion.

Organization:

- Decentralized, OTC market, with brokers and dealers.
- Small issues may be traded in exchanges.
- Daily trading volume in the U.S.: USD 822 billion (in 2012)
- Government debt dominates the market ( $50 \%$ of market).
- Non-financial corporations make up $12 \%$ of market ( $61 \%$ by U.S. corporations).
- Used to indicate the shape of the yield curve.

Note: The bond market is a public debt market, strictly regulated by a local body -in the U.S., the S.E.C. There is a private debt market: bank debt, private placements, and special debt vehicles (in the U.S., the popular Rule 144A debt).

## - The World Bond Market: Segments

- Domestic: A domestic firm issues local debt in local currency (70\% of World Bond Mkt). For example, Apple, the U.S. tech giant, issues USD bonds in the U.S. market.
- Foreign: A foreign firm issues local debt in the local currency. Regulated like a domestic issue. For example, Apple issues AUD bonds in the Australian market.
- Euro-yyy bond: A firm issues debt in a currency (yyy) different than the local currency where the bond is issued/trades. Domestic investors (from the firm's home country) are not the target of the issue. For example, Apple issues EUR bonds in the U.K market.

The Foreign + Eurobond Markets form the International Bond Market (30\% of World Bond Mkt).

According to Henderson et al. (2006), 20\% of corporate bonds are issued in the international bond market (from 1990-2001, USD 4.2 trillion). BTW, it is a high number relative to the $6 \%$ of public equity offering issued by corporations outside the home country.

Main currencies in the International Market (in Sep 2013): EUR (44\% of issues), USD (37\%), and GBP (9\%). The majority of the issues are straight (fixed rate) bonds (71\%).

## - Eurobond Market

Characteristics:

- Unregulated
- Bearer bonds (you hold them, you own them)
- AAA or governments borrow in Eurobond market
- Fastest way to raise funds for reputable borrowers
- Very creative market, with lots of instruments (straight bonds, FRN, dual zero-coupon bonds, currency bonds, convertibles, bonds with warrants, etc.)
- Most common bonds: Straight bonds (market share around 70\%).

Coupon payments for straight bonds are annual (YTM should be in p.a. basis). Coupon payments for floating bonds are semi-annual (YTM should be in s.a. basis).

## 2. Brief Review

There is a huge variety of bonds, loosely divided in straight or fixed rate, floating rate, and equity-related. In the international bond market, the majority of the issues are straight (fixed rate) bonds, with $71 \%$ of the market, while floating rate bonds have $26 \%$ of the market.

## 1.A Straight or Fixed Income Bonds

A fixed income bond is a financial instrument with specific interest payments on specified dates over a period of years. On the last specified date, or maturity, the payment includes a repayment of principal. The interest rate or coupon is expressed as a percentage of the issue amount and is fixed at launch.

Example: Straight bond.
In January 2004, the Brazilian Companhia Vale do Rio Doce (CVRD) issued straight coupon Eurobonds, with the following terms:

Amount: USD 500 million.
Maturity: January 2034 (30 years).
Issue price: 100 (or 100\%)
Coupon: $\quad 8.25 \%$ payable annually
YTM: $\quad 8.35 \%$ (Brazil's government bonds traded at YTM 9.02\%)
Note: CVRD, which is the world's largest iron ore miner, was initially planning to sell USD 300 million worth of the bonds, but ended up placing USD 500 million thanks to strong demand that surpassed USD1 billion. $\mathbb{I}$

## 1.B. Floating Rate Notes (FRNs)

FRNs are similar in structure to straight bonds but for the interest base and interest rate calculations. The coupon rate is reset at specified regular intervals, normally 3 months, 6 months, or one year. The coupon comprises a money market rate (e.g., LIBOR) plus a margin, which reflects the creditworthiness of the issuer.

Example: FRNs ("floaters").
In January 2004, Mexico issued a USD Eurobond, with the following terms:
Amount: USD 1,000 million.
Maturity: January 2009 (5 years).
Issue price: 99.965
Coupon: $\quad 6$-mo LIBOR +70 bps payable semiannually.
At the time the notes were offered, 6-mo LIBOR was 3.64 percent. So for the first six months Mexico paid an interest at an annual rate of $4.64 \% ~(=3.64 \%+.70 \%)$

Afterward, at the end of each six-month period, the interest rates on the bonds are updated to reflect the current 6 -mo LIBOR rate for dollars. 9

## 2. Q: How are bonds priced?

Bonds typically trade in 1,000 increments of a given currency (say, USD or EUR) and are priced as a percentage of par values (100\%). The price of a bond is determined by computing the NPV of all future cash flows generated by the bond discounted at an appropriate interest rate -i.e., YTM. There is a one-to-one relation between the price of a bond (P) and the YTM of a bond.
$\mathrm{P}=\mathrm{C}_{1} /(1+\mathrm{YTM})+\mathrm{C}_{2} /(1+\mathrm{YTM})^{2}+\mathrm{C}_{3} /(1+\mathrm{YTM})^{3}+\ldots+\mathrm{C}_{\mathrm{T}} /(1+\mathrm{YTM})^{\mathrm{T}}$,
$\mathrm{C}_{\mathrm{t}}=$ Cash flows the bond pays at time t . $\left(\mathrm{C}_{\mathrm{T}}=\right.$ coupon $_{\mathrm{T}}+$ Face Value $\left._{\mathrm{T}}\right)$
Once you know the YTM, you know the price -given that you know the coupon payments.
Interesting mathematical fact: If $\mathrm{C}=\mathrm{YTM} \quad \Rightarrow \mathrm{P}=100$ (par or 100\%)
Example: A straight Eurodollar bond matures in 1 year.

$$
\begin{aligned}
& \mathrm{C}=10 \% \\
& \mathrm{FV}=\text { USD } 100 \\
& \mathrm{P}=95 \text { (USD } 95) . \\
& \mathrm{YTM}_{\text {Bond }}=? \\
& \quad \mathrm{P}=\frac{\mathrm{C}+\mathrm{FV}_{1}-}{1+\mathrm{YTM}} \quad \Rightarrow 95=110 /(1+\mathrm{YTM}) . \\
& \quad \mathrm{YTM}=110 / 95-1 \quad \Rightarrow \mathrm{YTM}=.1578947
\end{aligned}
$$

## 3. Setting the YTM -i.e., the price- of a (Euro)bond

We have three main cases:
(1) Established company with a history of borrowing (say, GE, IBM);
(2) Established company with no history of borrowing (until 2010: MSFT, Google);
(3) New company.

## 1. Established company with history of borrowing

Example: IBM wants to borrow USD 100M in the next 10 days => Eurobonds
$\mathrm{YTM}_{\text {IBM }}=$ ?

- Look at competitors (by industry and credit rating).
- Look at secondary market (the best way).

IBM has outstanding bonds trading in the secondary market. $\mathrm{YTM}_{\text {IBM-outsanding }}=5.45 \%$
Then, $\mathrm{YTM}_{\text {IBM-new debt }}=5.45 \%$.

## 2. Established company with no history of borrowing

- Analyze the company.
- Look at competitors and industry benchmarks $\Rightarrow$ set a range, say YTM $\in$ [2.55\%, 4.10\%].
- Based on your analysis, pick a YTM in the range.


## 3. New company

Now, if a MNC is new to the Eurobond market, setting the YTM is more complicated.

- Look at competitors and industry benchmarks.
- Analyze the firm.
- Determine potential demand. Book building for the new bond (phone calls, lots of research).

The YTM is determined by:
YTM = Base Rate $\left(\mathrm{k}_{\mathrm{f}}\right)+$ Spread (Risk of Company)
$\mathrm{k}_{\mathrm{f}}=$ risk free rate $=$ government bond (of similar maturity)
Spread = Risk of company = this is what the investment bank has to determine (in bps)
In general, the spread is related to a risk rating (S\&P, Moody's). If a company is in a given risk category, there's a corresponding risk spread.

Example: Space Tourism (or an internet company in 1994, or railroads in the 1800s!) New company, no similar borrower in the market.

The investment banker determines that the YTM spread is in the range 140 bps to 210 bps over U.S. Treasuries ( $\mathrm{k}_{\mathrm{f}}$ ).

Aggressive spread: 140bps ( $\Rightarrow$ risk of not selling enough bonds -i.e., overpricing risk). Conservative spread: 210 bps ( $\Rightarrow$ risk of overselling bond issue -i.e., underpricing risk!). The investment banker decides on setting the Yield spread at 145 bps .

The lead manager is able to formulate a pricing scheme:
U.S. Treasury: $\quad 5.915 \%$ s.a. (semiannual)

ST spread: $\quad 1.45 \%$ s.a.
ST yield: $\quad 7.365 \%$ s.a. $\left(\Rightarrow\right.$ p.a. $\left.=(1+0.07365 / 2)^{2}-1\right)$
Technical detail:
Straight Eurobonds pay annual coupon. $\mathrm{YTM}_{\mathrm{ST}}=(1+0.07365 / 2)^{2}=7.501 \%$ p.a. (annual)
At inception, the bond sells at par $\quad \Rightarrow \mathrm{P}=100$ (if $\mathrm{P}=100 \Rightarrow \mathrm{C}=\mathrm{YTM}$ ).
Then, $\mathrm{C}_{\text {ST }}=\mathrm{YTM}=7.50 \%$. ■

## CHAPTER 18 - BONUS: ISSUING BOND DEBT - PETROBRAS

## Brazil's Petrobras breaks drought in LatAm market

Reuters January 9, 2017
By Paul Kilby

NEW YORK, Jan 9 (IFR) - Brazil's Petrobras opened the Latin American primary markets on Monday with the region's first cross-border bond sale of the year.

Petrobras is approaching investors with five and 10-year bonds to finance an up to US\$2bn tender as part of the debt-laden company's efforts to term out upcoming maturities.

At initial price thoughts of $6.5 \%$ area on the five-year and $7.75 \%$ area on the 10 , bankers are calculating starting new issue premiums at anywhere between 50bp and 75bp.

That calculation depends in part on how much weight to give the high dollar price on the existing $8.375 \%$ 2021s and $8.75 \%$ 2026s, which according to one banker were trading last week around 109 and 110.50 , or yields of $5.95 \%$ and $7.18 \%$.
"The dollar price is worth something but not worth 25bp," said one, arguing that investors like Petrobras bonds for their high liquidity.
"Liquidity means more than the high dollar price."
But other market participants disagreed. "I would say that fair value is close to $5.75 \%$ on the five year," another banker said. "The high dollar price matters a lot for this."

Either way, Petrobras is starting with a generous premium to get investors on board, much as Mexican state-controlled oil company Pemex did in December, when it amassed an over USD 30bn order book after initially offering lavish NICs.
"Petrobras isn't in a position these days where it can skin every last basis point on their deals," said the first banker.

The company has been struggling to regain its footing following a corruption scandal that has had a broad impact on Brazil's political and business classes.

New management has been shedding assets and undergoing liability management exercise in an effort to deleverage and improve the company's credit standing.

While Petrobras fell short of its 2015-2016 divestment target to raise USD 15.1bn, investors largely feel the company is heading in the right direction, and it is still seen as cheap to the sovereign.

In the tender, Petrobras is targeting USD-denominated 3\% 2019s, floating-rate 2019s, 7.875\% 2019s, $5.75 \%$ 2020s, $4.875 \%$ 2020s and floating-rate 2020s.

If holders tender by the early bird date of January 23, they will receive a purchase price of $100.625,101.625,110.50,104.875,102.75$ and 101.625 , respectively.

Petrobras is also offering to buy back euro-denominated 3.25\% 2019s at 105.125 if holders tender by the early bird date.

The bond is set to price on Monday. Bradesco, Citigroup, HSBC, Itau and Morgan Stanley are acting as leads. Expected ratings on the SEC-registered notes are B2/B+ (stable/negative).
(Reporting by Paul Kilby; Editing by Marc Carnegie)

## CHAPTER 18 - BONUS COVERAGE: BOND SPREADS (2014) REUTERS CORPORATE BOND SPREAD TABLES

| Reuters Corporate Spreads for Industrials - 03/28/2014 |  |  |  |  |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Rating | $\mathbf{1} \mathbf{~ y r}$ | $\mathbf{2} \mathbf{~ y r}$ | $\mathbf{3} \mathbf{~ y r}$ | $\mathbf{5} \mathbf{~ y r}$ | $\mathbf{7} \mathbf{~ y r}$ | $\mathbf{1 0} \mathbf{~ y r}$ | $\mathbf{3 0} \mathbf{~ y r}$ |  |
| Aaa/AAA | 5 | 8 | 12 | 18 | 28 | 42 | 65 |  |
| Aa1/AA+ | 10 | 18 | 25 | 34 | 42 | 54 | 77 |  |
| Aa2/AA | 14 | 29 | 38 | 50 | 57 | 65 | 89 |  |
| Aa3/AA- | 19 | 34 | 43 | 54 | 61 | 69 | 92 |  |
| A1/A+ | 23 | 39 | 47 | 58 | 65 | 72 | 95 |  |
| A2/A | 24 | 39 | 49 | 61 | 69 | 77 | 103 |  |
| A3/A- | 32 | 49 | 59 | 72 | 80 | 89 | 117 |  |


| Baa1/BBB+ | 38 | 61 | 75 | 92 | 103 | 115 | 151 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Baa2/BBB | 47 | 75 | 89 | 107 | 119 | 132 | 170 |
| Baa3/BBB- | 83 | 108 | 122 | 140 | 152 | 165 | 204 |
| Ba1/BB+ | 157 | 182 | 198 | 217 | 232 | 248 | 286 |
| Ba2/BB | 231 | 256 | 274 | 295 | 312 | 330 | 367 |
| Ba3/BB- | 305 | 330 | 350 | 372 | 392 | 413 | 449 |
| B1/B+ | 378 | 404 | 426 | 450 | 472 | 495 | 530 |
| B2/B | 452 | 478 | 502 | 527 | 552 | 578 | 612 |
| B3/B- | 526 | 552 | 578 | 604 | 632 | 660 | 693 |
| Caa/CCC+ | 600 | 626 | 653 | 682 | 712 | 743 | 775 |
| US Treasuries | 0.13 | 0.45 | 0.93 | 1.74 | 2.31 | 2.73 | 3.55 |

Spread values represent basis points (bps) over a US Treasury security of the same maturity, or the closest matching maturity.

## CHAPTER 18 - BONUS: BOND MATH -Bond Prices and Yields

Before reviewing the basics of bond mathematics, we will introduce some notation:
P = current market value of a bond = present value of the bond.
$\mathrm{FV}=$ face value or future value at a certain date, usually maturity.
$r=$ internal rate of return (yield-to-maturity) of a bond.
$\mathrm{T}=$ number of years to final maturity.
$\mathrm{C}=$ coupon rate of the bond.

## A.1. Yield-to-maturity

The theoretical value of a bond is determined by computing the present value of all future cash flows generated by the bond discounted at an appropriate interest rate. Conversely, one may calculate the internal rate of return, or yield-to-maturity (YTM), of a bond on the basis of its current market price and its promised payments. The YTM is also referred as yield. The YTM measures the expected total return on the overall investment. No other financial instrument has such an easily observed or intuitively understandable expected return.

Example: A straight Eurodollar bond has a coupon payment of 5\%. The market price of the Eurodollar bond is $\mathrm{P}=103.91$. Maturity is one year from now. The Eurodollar bond has a yield-to-maturity r, given by

$$
\mathrm{P}=\frac{\mathrm{C}+\mathrm{FV}_{1}-}{1+\mathrm{r}} \Rightarrow 103.91=100 /(1+\mathrm{r})
$$

Hence,

$$
103.91(1+r)=105 \Rightarrow(1+r)=105 / 103.91 \Rightarrow r=.0104898 .
$$

Similarly, one may compute the yield-to-maturity of zero-coupon bonds maturing in T years using the formula:

$$
\mathrm{P}=\frac{\mathrm{FV}_{\mathrm{T}}}{(1+\mathrm{r})^{\mathrm{T}}}
$$

where $r$ is expressed as a yearly interest rate. The term $1 /(1+r)^{T}$ is the discount factor for year T. The YTM is defined as the interest rate at which $P$ dollars should be invested in order to realize $F_{T}$ dollars $T$ years from now:

$$
\mathrm{P}(1+\mathrm{r})^{\mathrm{T}}=\mathrm{FV}_{\mathrm{T}}
$$

Example: A two-year zero-coupon Eurodollar bond paying FV ${ }_{2}=$ USD 100 is currently selling at a price P = USD 85.20, has a YTM, r, given by

$$
85.20=100 /(1+r)^{2}, \quad \Rightarrow r=(100 / 85.20)^{1 / 2}-1, \Rightarrow r=0.08338 . ~ \llbracket
$$

The YTM should not be confused with the current yield or dividend yield. The current yield on a bond is the ratio of the coupon bond to its current price.

Example: A bond with a price of 90 and a coupon of 10 percent has a current yield of:
$10 / 90=.1111=11.11 \%$. ब

## A.2. Yield curves

Similar bonds -i.e., bonds with similar characteristics: risks, coupons, maturities- should have the same return. Suppose we have bonds with similar characteristics but with the only exception of maturity. Graphing the yields to maturity on similar bonds with different maturities allows us to draw a yield curve. As illustrated below in Figure 18.1, the YTM of two zero-coupon bonds in the same currency but with different maturities is usually different.

Figure 18.1: Example of a Yield Curve from zero-coupon bonds


The yield curve shows the YTM computed on a given date as a function of the maturity of the bonds. Therefore, the graph should have on the horizontal or x -axis years to maturity and on the vertical or y -axis the YTM. It provides an estimate of the current term structure of interest rates. To be meaningful a yield curve must be drawn from bonds with identical characteristics, except for their maturity. In the graph, two zero-coupon bonds are represented as two points on the yield curve.

A yield curve is a best fit average of the individual yields, so the individual bond yields may well lie above or below the line when it is drawn. This gives an indication as to whether a particular bond has a relatively high or low yield in relation to its market. Points above the curve may be considered as highyielding (cheap), and those below as low-yielding (dear).

The normal slope of the curve when the market is in equilibrium is positive, that is, yields rise as maturity lengthens. Under these conditions, investors are receiving higher remuneration for forgoing immediate consumption and for the increased risks associated with longer-term investments.

The slope of the curve is important: the curve becomes steeper when the market expects a general rise in interest rates, and therefore traders sell longer dated bonds, forcing the price down and the yield up.

A flat curve arises when investors are indifferent to maturity risk, that is, short-term and long-term interest rates are very similar.

An uncommon yield curve arises where the yield curve has one or more humps of relative high yields, with lower yields on either side. Usually, there is a technical explanation for such a curve, such as oversupply of issues in a particular maturity band, for example a major issue by a government.

The so-called Treasury yield curve is constructed from on-the-run Treasury issues. These are the most recently auctioned Treasury securities: 3-month, 6-month, 1 -year, 2 -year, 4 -year, 5 -year, 10 -year, and 30year.

The first three issues are Treasury bills, which are issued at discount and pay no coupon. That is, these Treasury bills are zero-coupon securities. In contrast, the five other issues are coupon bonds. In fact, there are no zero-coupon securities issued by the U.S. Department of the Treasury with a maturity greater than one year. Consequently, the Treasury yield curve is a combination of zero-coupon securities and coupon securities.

There are, in fact, zero-coupon Treasury securities with a maturity greater than one year that are created by government dealer firms. These securities are called stripped Treasury securities. All stripped Treasury securities are created by dealer firms under a Treasury Department program called STRIP (Separate Trading of Registered Interest and Principal Securities).

## A.3. Valuing a bond with coupons

The theoretical value of a bond with coupons may be considered the present value of a stream of cash flows (coupons and principal payments). The cash flows occur at different times and they should be discounted at the interest rate corresponding to their date of disbursement. In essence, a coupon-paying bond is a combination of bonds with different maturities.

Example: A five-year EUR 1,000 bond with an annual coupon of EUR 80 is a combination of five bonds. Each bond has a nominal value of EUR 80 and a maturity of one to five years, and a bond with a nominal value of EUR 1,000 and a maturity of five-years. ๆ

The YTM, r, of a T-year coupon-paying bond is given by
$\mathrm{P}=\mathrm{C}_{1} /(1+\mathrm{r})+\mathrm{C}_{2} /(1+\mathrm{r})^{2}+\mathrm{C}_{3} /(1+\mathrm{r})^{3}+\ldots+\mathrm{C}_{\mathrm{T}} /(1+\mathrm{r})^{\mathrm{T}}$,
where $\mathrm{C}_{\mathrm{i}}$ is the coupon payment, including final reimbursement, at date $\mathrm{i}=\mathrm{T}$.

## $\bullet$ Price of a bond = $f($ Coupon, YTM, T)

The formula for the price of a bond shows that the bond's price is a function of the maturity of the coupon rate and of the YTM. Other factors being constant, the higher the coupon rate, the higher the value of the bond. Other factors being constant, the higher the YTM, the lower the price of the bond.

Coupons may be paid semiannually or quarterly, and a valuation may be made at any time during the coupon period. This calls for the more general valuation formula to determine YTM:
$\mathrm{P}=\mathrm{C}_{1} \mathrm{t}_{1} /(1+\mathrm{p})^{\mathrm{t} 1}+\mathrm{C}_{2} \mathrm{t}_{2} /(1+\mathrm{p})^{\mathrm{t} 2}+\mathrm{C}_{3} \mathrm{t}_{3} /(1+\mathrm{p})^{\mathrm{t} 3}+\ldots+\mathrm{C}_{\mathrm{T}} \mathrm{t}_{\mathrm{T}} /(1+\mathrm{p})^{\mathrm{tT}}$,
where p is the daily yield, i.e., $(1+\mathrm{p})^{365}=(1+\mathrm{r})$, and $\mathrm{t}_{1}, \mathrm{t}_{2}, . ., \mathrm{t}_{\mathrm{T}}$ are the dates on which the cash flows occur, expressed in number of days from the current date. Cash flows include all payments.

The majority of U.S. domestic bonds pay interest twice a year. In this case, the above formula simplifies to:
$\mathrm{P}=\mathrm{C}_{1} /(1+\mathrm{r} / 2)^{1}+\mathrm{C}_{2} /(1+\mathrm{r} / 2)^{2}+\mathrm{C}_{3} /(1+\mathrm{r} / 2)^{3}+\ldots+\mathrm{C}_{\mathrm{T}} /(1+\mathrm{r} / 2)^{\mathrm{T}}$,
where $r$ is the annual YTM of the bond.
Example: Annual v. Semiannual coupon bonds
A. Annual coupon bond.

An $8 \%$ annual coupon bond with a face value of USD 1,000 and with 5 years remaining to maturity has a YTM of $10 \%$, the current price $P$ is:
$\mathrm{P}=80 /(1.10)+80 /(1.10)^{2}+80 /(1.10)^{3}+80 /(1.10)^{4}+1,080 /(1.10)^{5}=924.18($ or $92.418 \%)$
Note: the YTM of $10 \%$ is higher than the current yield of USD 80/USD $924.18=8.66 \%$.
B. Semiannual coupon bond.

An $8 \%$ semi-annual coupon bond with a face value of USD 1,000 and with 5 years remaining to maturity has a YTM of $10 \%$, the current price P is:

$$
\begin{aligned}
P= & 40 /(1.05)+40 /(1.05)^{2}+40 /(1.05)^{3}+40 /(1.05)^{4}+40 /(1.05)^{5}+40 /(1.05)^{6}+ \\
& +40 /(1.05)^{7}+40 /(1.05)^{8}+40 /(1.05)^{9}+1040 /(1.05)^{10}=922.78 .
\end{aligned}
$$

Note: The YTM on the semiannual coupon bond is higher than the YTM on the annual bond. The annualized YTM on the semiannual yield is $(1+.10 / 2)^{2}-1=.1025$, or $10.25 \%$. ๆ

## A.4. Implied forward exchange rates

How do we compare exchange rate movements and YTM differentials? A higher yield in one currency is often compensated, ex post, by a depreciation in this currency, and in turn, an offsetting currency loss on the bond. It is important to know how much currency movement will exactly compensate the yield differential.

Consider a one-year T-bill with an interest rate $\mathrm{r}_{\mathrm{d}, 1}$ in domestic currency, and $\mathrm{r}_{\mathrm{f}, 1}$ in foreign currency. The current exchange rate is $\mathrm{S}_{\mathrm{t}}$, expressed as the domestic currency value of one unit of a foreign currency. Using the IRPT (interest rate parity theorem), derived in Chapter 7, it is possible to calculate the forward exchange rate, $\mathrm{F}_{\mathrm{t}, 1}$ that makes an investor indifferent between the two investments:

$$
1+\mathrm{r}_{\mathrm{d}, 1}=\left(1+\mathrm{r}_{\mathrm{f}, 1}\right) \mathrm{F}_{\mathrm{t}, 1} / \mathrm{S}_{\mathrm{t}} .
$$

The implied offsetting currency depreciation is given by

$$
\Delta \mathrm{S}_{1}=\mathrm{p}=\left(\mathrm{F}_{\mathrm{t}, 1}-\mathrm{S}_{\mathrm{t}}\right) / \mathrm{S}_{\mathrm{t}}=\left(\mathrm{r}_{\mathrm{d}, 1}-\mathrm{r}_{\mathrm{f}, 1}\right) /\left(1+\mathrm{r}_{\mathrm{f}, 1}\right) .
$$

You should notice that the implied offsetting change in $S_{t}$ was called, in Chapter 7, foreign currency premium (p).

Example: The USD one-year interest rate is $\mathrm{r}_{\mathrm{d}, 1}=5.468 \%$, the EUR interest rate is $\mathrm{r}_{\mathrm{f}, 1}=4.120 \%$, and the exchange rate is $\mathrm{S}_{\mathrm{t}}=1.10$ USD/EUR. The forward exchange rate is equal to:
$\mathrm{F}_{\mathrm{t}, 1}=\mathrm{S}_{\mathrm{t}}\left(1+\mathrm{r}_{\mathrm{d}, 1}\right) /\left(1+\mathrm{r}_{\mathrm{f}, 1}\right)=1.10$ USD/EUR $(1.05468) /(1.0412)=1.1142$ USD/EUR.
The foreign currency premium (or implied offsetting currency movement) is therefore equal to:
$\Delta \mathrm{S}_{1}=\mathrm{p}=(1.1142-1.10) /(1.10)=0.01295$ (or 1.295\%).
Thus, a $1.295 \%$ appreciation of the EUR will exactly compensate the yield advantage of the USD investment. $\boldsymbol{\|}$

Similarly, we can calculate implied forward exchange rates on two-year zero-coupon bonds as well as on bonds of longer maturity.

The implied forward exchange rate for a $t$-year bond is given by:
$\mathrm{F}_{\mathrm{t}, \mathrm{T}} \mathrm{S}_{\mathrm{t}}=\left[\left(1+\mathrm{r}_{\mathrm{d}, \mathrm{T}}\right) /\left(1+\mathrm{r}_{\mathrm{f}, \mathrm{T}}\right)\right]^{\mathrm{T}}$.
The implied currency appreciation or depreciation over the t-year period is equal to
$\Delta \mathrm{S}_{\mathrm{T}}=\left[\left(1+\mathrm{r}_{\mathrm{d}, \mathrm{T}}\right) /\left(1+\mathrm{r}_{\mathrm{f}, \mathrm{T}}\right)\right]^{\mathrm{T}}-1$.
Example: We are given two hypothetical term structures (five first years). Suppose that $\mathrm{S}_{0}=1.10$ USD/EUR. With this information we calculate the implied forward exchange rates in Table 18.1

Table 18.1: Implied Forward Exchange Rates and Currency Depreciation

| Maturity (T) | 1 | 2 | 3 | 4 | 5 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| EUR Yield (\%) | 4.120 | 4.322 | 4.544 | 4.678 | 4.792 |
| USD Yield (\%) | 5.468 | 5.618 | 5.645 | 5.658 | 5.850 |
| $\mathrm{~F}_{\mathrm{t}, \mathrm{T}}$ (USD/EUR) | 1.1142 | 1.1275 | 1.1351 | 1.1418 | 1.1567 |
| $\Delta \mathrm{~S}_{\mathrm{T}}(\%)$ | 1.295 | 2.500 | 3.193 | 3.798 | 5.151 |

For the above calculations, we assume that the yield curves are for zero-coupon bonds. The formulas are slightly more complicated if we use yield curves for coupon bonds, because we must assume that the coupons are reinvested each year or semester until final maturity.

